$\boxed{\text{Day 1 - AM}}$

Logic and Proofs

Main Structures

- implication $(A \Rightarrow B)$
  if A is true, B must be as well
  if A is false, we don't know about B

- AND $(A \wedge B)$
  true if and only if both A and B are true, false otherwise

- OR $(A \vee B)$
  true if at least one of A or B is true, false if both false

- converse $(B \Rightarrow A)$

- equivalence $(A \Leftrightarrow B)$
  means $A \Rightarrow B$ and $B \Rightarrow A$

Proof Techniques $A \Rightarrow B$

① Direct Proof: Assume A, show B

② Contradiction: Assume A and not B. Show incompatible

③ Contrapositive: Assume not B, Show not A.

④ Induction: Show works for base case, Deduce that if it works for n, it also works for n+1. →

Ex: Prove $\sqrt{2}$ is irrational

Pf: Assume $\sqrt{2}$ is rational.

Then $\sqrt{2} = \frac{p}{q}$ in lowest terms for some integers p, q

Then $\sqrt{2}\, q = p$

$\Rightarrow 2q^2 = p^2$ so p^2 is even so $p = 2r$ for some integer r

then $2q^2 = (2r)^2$

$\Rightarrow 2q^2 = 4r^2$

$q^2 = 2r^2$ so q^2 is even and q is even.

Thus p and q are even and thus share a factor of 2.

So $\frac{p}{q}$ not in lowest terms. $\Rightarrow\Leftarrow$ $\blacksquare$

In proofs...

"for all x": show that ANY choice of x works.

"there exists an x": produce a candidate and show it works.

Functions

$f: D \to C$, a rule assigning to each element in D some element of C.

domain: valid inputs

codomain: possible outputs

range: all actual outputs

for $x \in D$, $f(x) \in C$, $f(x)$ is called the image

Ex: $f(x) = x^2$ $f: \mathbb{R} \to \mathbb{R}$ domain: $\mathbb{R}$ codomain: $\mathbb{R}$

range: $[0, \infty)$

$\longrightarrow$

Limits

$$\lim_{x \to c} f(x) = L$$

for every $\varepsilon > 0$, there exists $\delta > 0$ such that $|x-c| < \delta$ implies $|f(x) - L| < \varepsilon$

We say $f(x)$ is Continuous if $\lim_{x \to c} f(x) = f(c)$